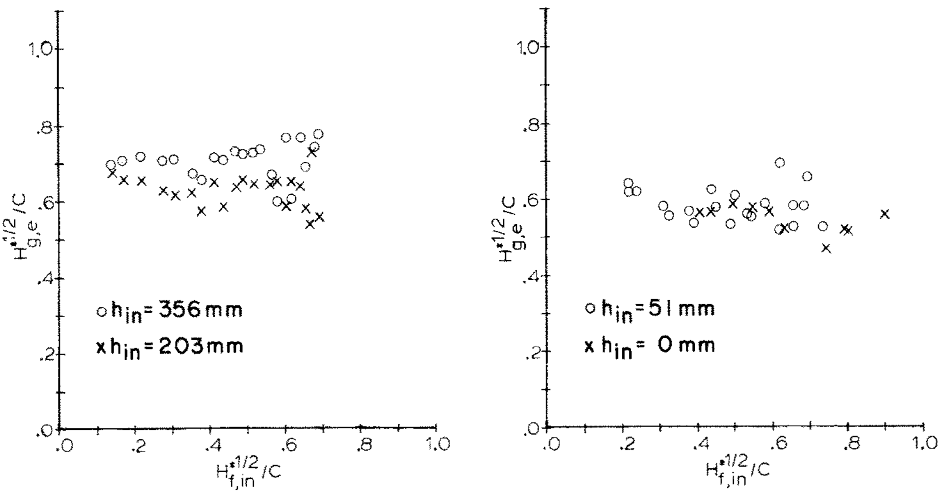

FIG. 4. Dimensionless steam flow rate for total-delivery point.

and zero penetration curves tended to intersect. Sudden collapses of the water pool were observed and the flow pattern switched back and forth between the two extremes. The data for 0 mm water injection [7] showed that even higher steam flow rates were required.

Equation (6) is used to correlate all the total delivery data and the no-delivery data for the two highest water inlet positions, 356 and 203 mm, and equation (9) is used for the no-delivery data of the two lowest water inlet positions, 51 and 0 mm. As shown in Fig. 3, the zero-penetration points are in good agreement with equation (8). Assuming that equation (6) is valid just before total dumping occurs, the complete-penetration data presented in Fig. 4 show a minimum value for the dimensionless steam flow rate parameter of about 0.6, which is independent of the value of the dimensionless water flow rate parameter.

It is therefore concluded that, for this particular rectangular geometry, intended to simulate the upper tie-plate of the German PKU reactor, vertical injection of cold water close to the tie-plate can be more efficient in cooling the reactor core, since delivery is observed for smaller water flow rates compared to horizontal spray experiments [3]. However, the importance of the condensation efficiency, f, for other geometries, as well as the effects of liquid subcooling, jet diameter and jet velocity at the plate, are subjects for further investigation.

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Int. J. Heat Mass Transfer. Vol. 28, No. 12, pp. 2385–2388, 1985
Printed in Great Britain
0017-9310/85 $3.00 + 0.00

Similarity between unsteady conduction and natural convection

M. G. DAVIES
Department of Building Engineering, The University, Liverpool L69 3BX, U.K.

(*Received* 20 *March* 1985 *and in final form* 26 *June* 1985)

1. INTRODUCTION

THE TRANSFER of heat by unsteady conduction through a solid—the wall of a room for example—and the process of natural convection that typically takes place between the air and the wall of a room, are physically speaking very dissimilar processes. Nevertheless, the analytical expressions for the heat flow at the surface due to the processes show some structural similarities as will be demonstrated below.

2. SINUSOIDALLY DRIVEN CONDUCTED HEAT FLOW

One-dimensional heat flow in a solid is subject to the equation

$$k\frac{\partial^2 T}{\partial x^2} = \rho c\frac{\partial T}{\partial t}. \tag{1}$$

NOMENCLATURE

Constants
- k conductivity
- ρ density
- c specific heat
- μ viscosity
- β coefficient of expansion
- g acceleration due to gravity.

Groups
- h convection coefficient
- τ cyclic thickness, $(\pi\rho cX^2/Pk)^{1/2}$
- a characteristic admittance, $(2\pi k\rho c/P)^{1/2}$
- Nu Nusselt number, hl/k
- Pr Prandtl number, $\mu c/k$
- Gr Grashof number, $\rho^2 l^3 \beta\theta g/\mu^2$
- S $\sqrt{l/\beta\theta g}$
- J $h/(k\rho c/S)^{1/2}$.

Imposed and resulting variables
- l plate height
- X slab thickness
- P periodic time
- T temperature
- θ temperature difference
- q heat flow.

If the heat flow is associated with sinusoidal variation of a surface temperature, then the form of the temperature profile in a semi-infinite solid is:

$$T(x,t) = T_s \exp(-\alpha x)\exp(-j\beta x)\exp(j2\pi t/P). \quad (2)$$

The four right-hand terms denote, respectively, the amplitude of variation of temperature at the surface, the damping of the wave in the solid, a superposed standing wave form and the time variation.

Equations (1) and (2) are consistent if

$$\alpha = \beta = \sqrt{\frac{\pi\rho c}{Pk}}. \quad (3)$$

The heat flow is given by

$$q(x,t) = -k\frac{\partial T(x,t)}{\partial x}. \quad (4)$$

It follows that the ratio of amplitudes of heat flow to temperature at the surface $x = 0$ is

$$\mathbf{a} = \frac{q_s}{T_s} = \frac{q(0,t)}{T(0,t)} = \sqrt{\frac{2\pi k\rho c}{P}}\exp(j\pi/4). \quad (5)$$

Here q_s and T_s are, respectively, the amplitudes of heat flux and temperature at the surface. The equation shows that q_s peaks 1/8 of a cycle or 3 h in 24 h after T_s. $\mathbf{a}$ is the 'characteristic admittance' for the material. For brickwork excited at the diurnal period, a, the magnitude of $\mathbf{a}$, is around $10\ \mathrm{W\,m^{-2}\,K^{-1}}$.

If the solid is instead bounded by two parallel planes distant X apart, the temperatures and heat flows at surfaces 1 and 2 are related by the transmission matrix for the slab (see ref. [1]);

$$\begin{bmatrix} T_1 \\ q_1 \end{bmatrix} = \begin{bmatrix} \cosh(\tau + j\tau) & \sinh(\tau + j\tau)/\mathbf{a} \\ \sinh(\tau + j\tau)\mathbf{a} & \cosh(\tau + j\tau) \end{bmatrix} \begin{bmatrix} T_2 \\ q_2 \end{bmatrix} \quad (6)$$

where τ is the 'cyclic thickness', given by

$$\tau^2 = \frac{\pi\rho cX^2}{Pk} \quad (7)$$

and is dimensionless. If τ is small, less than 0.1 say, the layer is thermally thin in the sense that it does not sustain a large temperature difference between its faces. For a single thickness of brick, τ is around unity. If τ is greater than about 3, as far as surface 1 is concerned, the slab is effectively infinitely thick.

Two boundary conditions at surface 2 are of current interest.

2.1. *One surface adiabatic*

If surface 2 is perfectly insulated, $q_2 = 0$. It can be shown then that the magnitude of the ratio of q_s to T_s is given by

$$y_{sa} = \frac{q_s}{T_s} = \sqrt{\frac{2\pi k\rho c}{P}}\cdot\sqrt{\frac{\cosh 2\tau - \cos 2\tau}{\cosh 2\tau + \cos 2\tau}}. \quad (8)$$

(The phase relation between q_s and T_s can also be written down, but it is not relevant to the present problem.)

If the slab is thin, so that τ tends to zero, it appears that

$$\frac{q_s}{T_s} \to \frac{2\pi\rho cX}{P}. \quad (9)$$

2.2 *One surface isothermal*

If surface 2 is held at constant temperature, $T_2 = 0$. We then have comparable results:

$$y_{si} = \frac{q_s}{T_s} = \sqrt{\frac{2\pi k\rho c}{P}}\cdot\sqrt{\frac{\cosh 2\tau + \cos 2\tau}{\cosh 2\tau - \cos 2\tau}} \quad (10)$$

and when τ tends to zero

$$\frac{q_s}{T_s} \to \frac{k}{X}. \quad (11)$$

2.3. *Discussion*

The quantity q_s/T_s is similar in structure to a heat transfer coefficient. Its value depends upon the thickness and boundary condition imposed upon the slab. For the present purpose, its dependence can be described as follows:

(i) q_s/T_s tends to be proportional to the product $\sqrt{k\rho c}$. $\sqrt{k\rho c}$ represents the combined ability of a solid to conduct and to store heat.

(ii) q_s/T_s tends to be proportional to $\sqrt{1/P}$. P, the periodicity, is imposed here by external means, and the larger it is, the smaller (q_s/T_s) must become, as is physically obvious.

(iii) q_s/T_s further depends upon the quotient $k/\rho c$ which is a grouping appearing in τ, equation (7). $k/\rho c$ too has a physical interpretation. The standard form for the profile of a sinusoidal travelling wave is:

$$y = a\cos\left[\frac{2\pi}{\lambda}(c't - x)\right] \quad (12)$$

where y and a denote displacement and amplitude, λ is the wavelength and c' is the velocity of propagation. If the last two factors in equation (2) are combined, the real part of the wave-like behaviour of the temperature profile can be expressed by the term:

$$\cos\left[\left(2\pi\Big/\left(\frac{4\pi Pk}{\rho c}\right)^{1/2}\right)\left(\sqrt{\frac{4\pi k}{P\rho c}}\,t - x\right)\right]. \quad (13)$$

By comparison of the expressions (12) and (13), the velocity of propagation c' is equal to $(4\pi k/P\rho c)^{1/2}$; thus the quotient $\sqrt{k/\rho c}$ helps determine the rate of propagation of a thermal signal.

(iv) The dependence of q_s/T_s on k, ρ and c can be described as being proportional to $\rho^u\sqrt{k\rho c}$ where u ranges between $-\frac{1}{2}$ and $+\frac{1}{2}$ according to circumstances.

3. CONVECTIVELY DRIVEN HEAT FLOW

3.1. *Laminar flow*

Natural convection at a vertical wall or plate of height l, when laminar, can be described by an expression of form

$$Nu = A(Gr\cdot Pr)^{1/4}. \tag{14}$$

From observational data, A has a value of around 0.62. A can be found from a detailed mechanistic analysis [2, p. 520]:

for vanishingly small values of Pr, $A = 0.8\ Pr^{1/4}$;

for $Pr = \infty$, $A = 0.67$; for air, $A = 0.517$.

A somewhat simpler theoretical argument leads to an alternative form for Nu:

$$Nu = \left(\frac{512}{2430}\right)^{1/4}\cdot\left(\frac{Pr}{Pr+20/21}\right)^{1/4}\cdot(Gr\cdot Pr)^{1/4}. \tag{15}$$

It is useful to rewrite these equations so as to show directly the physical dependence of h, the convective heat transfer coefficient, upon the physical properties of the fluid. We write:

$$S = \sqrt{l/\beta\theta g}. \tag{16}$$

The values of h from equations (14) and (15) are, respectively

$$h = \frac{A}{Pr^{1/4}}\cdot\sqrt{k\rho c/S}. \tag{17}$$

and

$$h = \frac{0.68}{(Pr+20/21)^{1/4}}\cdot\sqrt{k\rho c/S}. \tag{18}$$

3.2. *Turbulent flow*

Empirical evidence on turbulent natural convection at a vertical surface leads to the relation

$$Nu = 0.12\,(Gr\cdot Pr)^{1/3} \tag{19}$$

and a theoretical argument leads to

$$Nu = \frac{0.0248\,Pr^{1/15}}{(1+0.494\,Pr^{2/3})^{2/5}}(Gr\cdot Pr)^{2/5}. \tag{20}$$

If the physical quantities are inserted into these equations we have:

$$h = \frac{0.12\,Gr^{1/12}}{Pr^{1/6}}\sqrt{k\rho c/S} \tag{21}$$

and

$$h = \frac{0.0248\,Gr^{3/20}}{(1+0.494\,Pr^{2/3})^{2/5}\,Pr^{1/30}}\sqrt{k\rho c/S}. \tag{22}$$

3.3. *Laminar flow at low pressures*

The equations for laminar flow suggest that the quantity $J = h/(k\rho c/S)^{1/2}$ should be constant during the stage of laminar flow. As part of this study, the data of Saunders [3] was reworked to examine this point. Saunders conducted tests on a number of small plates held at 70°C in air at 15°C, but with pressures ranging from 0.043 to 67 atm. (In reworking this data, Saunders' values for all parameters have been adopted. This includes his value of $\beta = 1/272$ at a mean layer temperature of 42°C or 315 K, and his value of $Pr = 0.78$ at all pressures, a value which is undoubtedly too high at low pressures. Since Saunders' results, together with other results in the form of $Nu\,(Gr\cdot Pr)$ correlations, were included in his book [4] and have been frequently quoted since, his results have been used here without alteration.)

J is presented as a function of log $\sqrt{Gr}$ in Fig. 1.

Between $\sqrt{Gr} = 10^3$ and $10^{4.7}$ or so, J is approximately equal to 0.6. This is in satisfactory agreement with the value of $0.68/(Pr+20/21)^{1/4}$ or 0.59. At $Gr = 10^{4.7}$, J starts to increase, corresponding to the onset of turbulence at the top of the plate. At this stage J depends upon Gr and the data indicate that J is proportional to $Gr^{0.11}$ or so. This is intermediate between the powers suggested in equations (21) and (22). This form of plot shows the transition to turbulence more clearly than the usual plot of log Nu against log Gr.

The data indicate a gradual decrease in J up to $\sqrt{Gr} = 10^3$. Churchill and Chu [6, equation (5) and Fig. 1] provide a

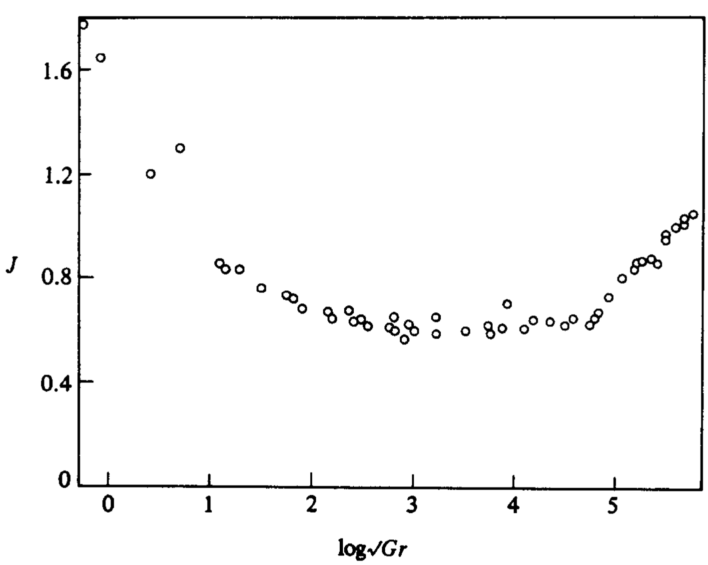

FIG. 1. J as a function of log $\sqrt{GR}$.

description which in effect adds the term 0.68 to the RHS of equation (15). They suggest that the additional term may be due to 'edge effects'. In Saunders' experiments the platinum strips forming the vertical plates were supported between "pairs of copper bars ... of sufficient cross section ..." by means of which a heating current was passed through the strip. The copper bars were presumably at the ambient temperature of 15°C and it may be that conduction losses from the platinum to the copper led to computation of spuriously high convective heat transfer coefficients. However, it may be noted that the heat transfer that takes place across a very narrow vertical cavity is dominated by air conduction; the net motion of the air described by the ρc term, is negligible. It may be that for convection at a vertical plate there is a similar, if small, residual effect due to conduction which is only apparent at very low Grashof numbers. According to Fig. 1 of ref. [5] the only data available at low values of $Gr \cdot Pr$ are those of Saunders, so it is not possible to decide between the end effect, the conduction, or any other mechanism to explain the anomalously high h values.

If Churchill and Chu's suggestion of a limiting value for h should have some real basis, it may be possible to express h at low pressures as something like:

$$h = 0.68\frac{k}{l}. \tag{23}$$

3.4. *Discussion*

It is apparent from the foregoing analysis that h too depends on certain groups:

(i) It tends to be proportional to $\sqrt{k\rho c}$. k describes the ability of the air or other fluid to transfer heat from the wall; ρc describes the ability of the stream parallel to the wall to remove the heat once it is in the stream.

(*ii*) h tends to be proportional to $\sqrt{1/S}$. Now according to the integral method of handling laminar natural convection, the mean velocity of the fluid at height l is

$$\bar{v} = \frac{5/27}{(Pr+20/21)^{1/2}}\sqrt{\beta\theta g l} = 0.33\sqrt{\beta\theta g l}$$

and so S, which is defined as $(l/\beta\theta g)^{1/2}$, is of order $1/\bar{v}$, or of order of the time taken for a particle entrained at the bottom of the plate to rise up the height of the plate.

(iii) h has a further dependence on Pr. Now Pr can be expressed as:

$$Pr = \frac{\mu/\rho}{k/\rho c}. \tag{24}$$

The numerator in this form, the kinematic viscosity, is concerned with velocity in fluid flow and the denominator (the diffusivity) with the propagation of temperature. Indeed, Ede [6, p. 74], remarks that Pr can be interpreted as "an indication of the relative rates at which velocity and temperature disturbances are propagated through a fluid".

(iv) The dependence of h upon k, ρ and c can be described as being proportional to $\rho^u\sqrt{k\rho c}$ where $u = 0$ for laminar flow and according to equation (22) might reach 0.3 for turbulent flow. For low pressures, it might tend to -0.5 but this is speculative.

4. COMPARISON BETWEEN THE PROCESSES

A comparison between the discussions of Sections 2 and 3 shows some common features in the expression for heat flow in the two cases.

(i) Both q_s/T_s and h depend upon the material properties in the form $\sqrt{k\rho c}$.

(ii) Both depend upon a time scale in the form $\sqrt{1/\text{time scale}}$. For sinusoidally varying conduction, the time scale P is imposed by the periodicity itself. For convection, the time scale comes about as a result of other quantities, l, θ, together with ρ and g, which can be regarded as the independent variables.

(iii) Both may depend upon a diffusivity grouping $k/\rho c$.

(iv) Both may depend upon a further factor, ρ^u, where u may be positive or negative.

Specific differences lie in the presence in the conduction case of the grouping X^2/P to form τ, and in the convection case of the grouping μ/ρ to form Pr. The common grouping of course is $k/\rho c$.

The functions that involve τ are quite unlike the functions that involve Pr.

5. DISCUSSION

The parallels have significance in a situation where heat may be transferred up to a surface by convection and away by conduction, which is often the case in building heat transfer. Useful quantities describing the behaviour of a wall construction including the convective transfer inside and outside (setting aside radiation for this discussion), are the parameters

$$u = \text{dynamic transmittance} = \frac{\text{heat flow from wall to inside air}}{\text{outside temperature}}$$

with inside air temperature held constant;

$$y = \text{admittance} = \frac{\text{heat flow to wall from inside air}}{\text{inside temperature}}$$

with outside air temperature held constant. In each case, the sinusoidal variation of heat flow and temperature is intended.

For a simple homogeneous wall flanked by films with heat transfer coefficients h_i and h_o, u and y are each functions of h_i, h_o, k, ρc, P, and X. Writing $a = \sqrt{2\pi k\rho c/P}$, u and y can be nondimensionalised as u/a and y/a. They will then be functions of further parameters which can be selected as (i) $\tau = \sqrt{\pi\rho c X^2/Pk}$ as noted earlier; (ii) h_i/h_o, obviously the ratio of two like quantities; and (iii) $a/\sqrt{h_i h_o}$, or a/h_i for an internal wall.

This last quantity is partly made up as the ratio $[(k\rho c)_{\text{solid}}/(k\rho c)_{\text{fluid}}]^{1/2}$, again the ratio of similar quantities.

Parameters in the present form were used by the present author in ref. [1] for compilation of Tables 5–8 therein.

A further example is to be found in transient heat flow: if the fluid wetting the surface of a semi-infinite slab suffers a step in temperature, the subsequent variation in surface temperature is to be expressed by time, non-dimensionalised as $h^2t/(k\rho c)$. The present author has used it in this form [7] for use in Tables 4 and 5.

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